13.

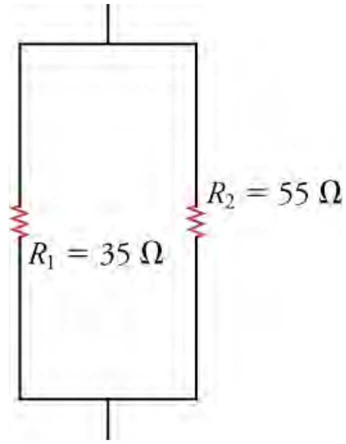


Figure 19.19

What is the equivalent resistance for the two resistors shown below?

a. The equivalent resistance is 20 Ω
b. The equivalent resistance is 21 Ω
c. The equivalent resistance is 90 Ω
d. The equivalent resistance is 1,925 Ω

Check Your Understanding

14. The voltage drop across parallel resistors is ________.

a. the same for all resistors
b. greater for the larger resistors
c. less for the larger resistors
d. greater for the smaller resistors

15. Consider a circuit of parallel resistors. The smallest resistor is 25 Ω . What is the upper limit of the equivalent resistance?

a. The upper limit of the equivalent resistance is 2.5 Ω.
b. The upper limit of the equivalent resistance is 25 Ω.
c. The upper limit of the equivalent resistance is 100 Ω.
d. There is no upper limit.

19.4 Electric Power

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define electric power and describe the electric power equation
- Calculate electric power in circuits of resistors in series, parallel, and complex arrangements

Section Key Terms

electric power

Power is associated by many people with electricity. Every day, we use electric power to run our modern appliances. Electric power transmission lines are visible examples of electricity providing power. We also use electric power to start our cars, to run our computers, or to light our homes. Power is the rate at which energy of any type is transferred; **electric power** is the rate at which electric energy is transferred in a circuit. In this section, we'll learn not only what this means, but also what factors determine electric power.

To get started, let's think of light bulbs, which are often characterized in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb (see Figure 19.20). Although both operate at the same voltage, the 60-W bulb emits more light intensity than the 25-W bulb. This tells us that something other than voltage determines the power output of an electric circuit.

Incandescent light bulbs, such as the two shown in Figure 19.20, are essentially resistors that heat up when current flows through them and they get so hot that they emit visible and invisible light. Thus the two light bulbs in the photo can be considered as two different resistors. In a simple circuit such as a light bulb with a voltage applied to it, the resistance determines the current by Ohm's law, so we can see that current as well as voltage must determine the power.

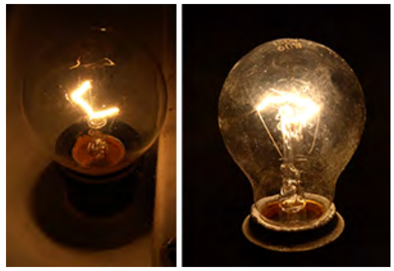


Figure 19.20 On the left is a 25-W light bulb, and on the right is a 60-W light bulb. Why are their power outputs different despite their operating on the same voltage?

The formula for power may be found by dimensional analysis. Consider the units of power. In the SI system, power is given in watts (W), which is energy per unit time, or J/s

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}. \tag{19.47}$$

Recall now that a voltage is the potential energy per unit charge, which means that voltage has units of J/C

$$\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}. \tag{19.48}$$

We can rewrite this equation as $\mathrm{J} = \mathrm{V} \times \mathrm{C}$ and substitute this into the equation for watts to get

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{V} \times \mathrm{C}}{\mathrm{s}} = \mathrm{V} \times \frac{\mathrm{C}}{\mathrm{s}}.$$

But a Coulomb per second (C/s) is an electric current, which we can see from the definition of electric current, $I = \frac{\Delta Q}{\Delta t}$, where ΔQ is the charge in coulombs and Δt is time in seconds. Thus, equation above tells us that electric power is voltage times current, or

$$P = IV.$$

This equation gives the electric power consumed by a circuit with a voltage drop of V and a current of I.

For example, consider the circuit in Figure 19.21. From Ohm's law, the current running through the circuit is

$$I = \frac{V}{R} = \frac{12\ \mathrm{V}}{100\ \Omega} = 0.12\ \mathrm{A}. \tag{19.49}$$

Thus, the power consumed by the circuit is

$$P = VI = (12\ \mathrm{V})(0.12\ \mathrm{A}) = 1.4\ \mathrm{W}. \tag{19.50}$$

Where does this power go? In this circuit, the power goes primarily into heating the resistor in this circuit.

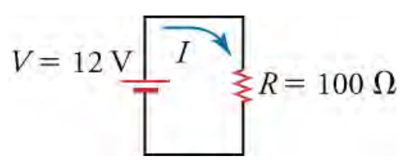


Figure 19.21 A simple circuit that consumes electric power.

In calculating the power in the circuit of Figure 19.21, we used the resistance and Ohm's law to find the current. Ohm's law gives the current: $I = V/R$, which we can insert into the equation for electric power to obtain

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}.$$

This gives the power in terms of only the voltage and the resistance.

We can also use Ohm's law to eliminate the voltage in the equation for electric power and obtain an expression for power in terms of just the current and the resistance. If we write Ohm's law as $V = IR$ and use this to eliminate V in the equation $P = IV$, we obtain

$$P = IV = I\,(IR) = I^2 R.$$

This gives the power in terms of only the current and the resistance.

Thus, by combining Ohm's law with the equation $P = IV$ for electric power, we obtain two more expressions for power: one in terms of voltage and resistance and one in terms of current and resistance. Note that only resistance (not capacitance or anything else), current, and voltage enter into the expressions for electric power. This means that the physical characteristic of a circuit that determines how much power it dissipates is its resistance. Any capacitors in the circuit do not dissipate electric power—on the contrary, capacitors either store electric energy or release electric energy back to the circuit.

To clarify how voltage, resistance, current, and power are all related, consider Figure 19.22, which shows the *formula wheel*. The quantities in the center quarter circle are equal to the quantities in the corresponding outer quarter circle. For example, to express a potential V in terms of power and current, we see from the formula wheel that $V = P/I$.

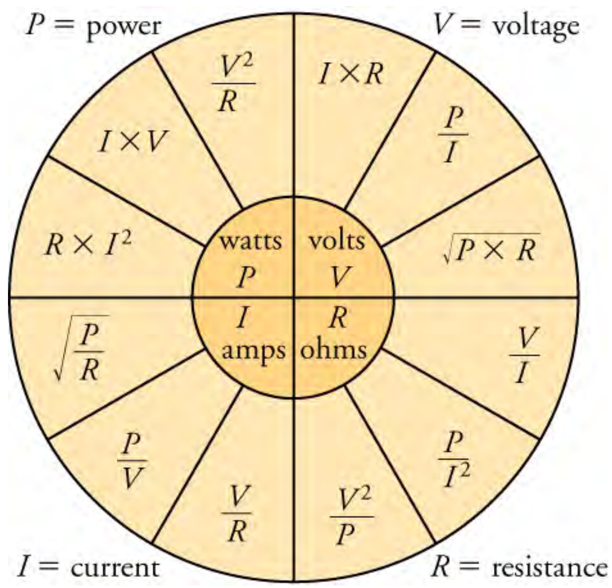


Figure 19.22 The formula wheel shows how volts, resistance, current, and power are related. The quantities in the inner quarter circles equal the quantities in the corresponding outer quarter circles.

WORKED EXAMPLE

Find the Resistance of a Lightbulb

A typical older incandescent lightbulb was 60 W. Assuming that 120 V is applied across the lightbulb, what is the current through the lightbulb?

STRATEGY

We are given the voltage and the power output of a simple circuit containing a lightbulb, so we can use the equation $P = IV$ to find the current I that flows through the lightbulb.

Solution

Solving $P = IV$ for the current and inserting the given values for voltage and power gives

$$\begin{aligned} P &= IV \\ I &= \frac{P}{V} = \frac{60\ \text{W}}{120\ \text{V}} = 0.50\ \text{A}. \end{aligned} \tag{19.51}$$

Thus, a half ampere flows through the lightbulb when 120 V is applied across it.

Discussion

This is a significant current. Recall that household power is AC and not DC, so the 120 V supplied by household sockets is an alternating power, not a constant power. The 120 V is actually the time-averaged power provided by such sockets. Thus, the average current going through the light bulb over a period of time longer than a few seconds is 0.50 A.

WORKED EXAMPLE

Boot Warmers

To warm your boots on cold days, you decide to sew a circuit with some resistors into the insole of your boots. You want 10 W of heat output from the resistors in each insole, and you want to run them from two 9-V batteries (connected in series). What total resistance should you put in each insole?

STRATEGY

We know the desired power and the voltage (18 V, because we have two 9-V batteries connected in series), so we can use the equation $P = V^2/R$ to find the requisite resistance.

Solution

Solving $P = V^2/R$ for the resistance and inserting the given voltage and power, we obtain

$$\begin{aligned} P &= \frac{V^2}{R} \\ R &= \frac{V^2}{P} = \frac{(18\ \text{V})^2}{10\ \text{W}} = 32\ \Omega. \end{aligned} \tag{19.52}$$

Thus, the total resistance in each insole should be 32 Ω.

Discussion

Let's see how much current would run through this circuit. We have 18 V applied across a resistance of 32 Ω, so Ohm's law gives

$$I = \frac{V}{R} = \frac{18\ \text{V}}{32\ \Omega} = 0.56\ \text{A}. \tag{19.53}$$

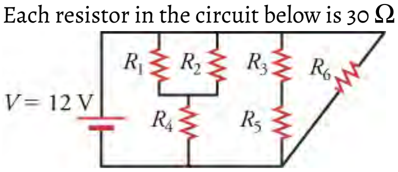
All batteries have labels that say how much charge they can deliver (in terms of a current multiplied by a time). A typical 9-V alkaline battery can deliver a charge of 565 $\text{mA} \cdot \text{h}$ (so two 9 V batteries deliver 1,130 $\text{mA} \cdot \text{h}$), so this heating system would function for a time of

$$t = \frac{1130\ \times\ 10^{-3}\,\text{A} \cdot \text{h}}{0.56\ \text{A}} = 2.0\ \text{h}. \tag{19.54}$$

WORKED EXAMPLE

Power through a Branch of a Circuit

Each resistor in the circuit below is 30 Ω . What power is dissipated by the middle branch of the circuit?



STRATEGY

The middle branch of the circuit contains resistors R_3 and R_5 in series. The voltage across this branch is 12 V. We will first find the equivalent resistance in this branch, and then use $P = V^2/R$ to find the power dissipated in the branch.

Solution

The equivalent resistance is $R_{\text{middle}} = R_3 + R_5 = 30\ \Omega + 30\ \Omega = 60\ \Omega$. The power dissipated by the middle branch of the circuit is

$$P_{\text{middle}} = \frac{V^2}{R_{\text{middle}}} = \frac{(12\ \text{V})^2}{60\ \Omega} = 2.4\ \text{W}. \tag{19.55}$$

Discussion

Let's see if energy is conserved in this circuit by comparing the power dissipated in the circuit to the power supplied by the battery. First, the equivalent resistance of the left branch is

$$R_{\text{left}} = \frac{1}{1/R_1 + 1/R_2} + R_4 = \frac{1}{1/30\ \Omega + 1/30\ \Omega} + 30\ \Omega = 45\ \Omega. \tag{19.56}$$

The power through the left branch is

$$P_{\text{left}} = \frac{V^2}{R_{\text{left}}} = \frac{(12\ \text{V})^2}{45\ \Omega} = 3.2\ \text{W}. \tag{19.57}$$

The right branch contains only R_6, so the equivalent resistance is $R_{\text{right}} = R_6 = 30\ \Omega$. The power through the right branch is

$$P_{\text{right}} = \frac{V^2}{R_{\text{right}}} = \frac{(12\ \text{V})^2}{30\ \Omega} = 4.8\ \text{W}. \tag{19.58}$$

The total power dissipated by the circuit is the sum of the powers dissipated in each branch.

$$P = P_{\text{left}} + P_{\text{middle}} + P_{\text{right}} = 2.4\ \text{W} + 3.2\ \text{W} + 4.8\ \text{W} = 10.4\ \text{W} \tag{19.59}$$

The power provided by the battery is

$$P = IV. \tag{19.60}$$

where I is the total current flowing through the battery. We must therefore add up the currents going through each branch to obtain I. The branches contributes currents of

$$\begin{aligned} I_{\text{left}} &= \frac{V}{R_{\text{left}}} = \frac{12\ \text{V}}{45\ \Omega} = 0.2667\ \text{A} \\ \text{I}_{\text{middle}} &= \frac{V}{R_{\text{middle}}} = \frac{12\ \text{V}}{60\ \Omega} = 0.20\ \text{A} \\ \text{I}_{\text{right}} &= \frac{V}{R_{\text{right}}} = \frac{12\ \text{V}}{30\ \Omega} = 0.40\ \text{A}. \end{aligned} \tag{19.61}$$

The total current is

$$I = I_{\text{left}} + I_{\text{middle}} + I_{\text{right}} = 0.2667\ \text{A} + 0.20\ \text{A} + 0.40\ \text{A} = 0.87\ \text{A}. \tag{19.62}$$

and the power provided by the battery is

$$P = IV = (0.87\ \text{A})(12\ \text{V}) = 10.4\ \text{W}. \tag{19.63}$$

This is the same power as is dissipated in the resistors of the circuit, which shows that energy is conserved in this circuit.

Practice Problems

16. What is the formula for the power dissipated in a resistor?
   a. The formula for the power dissipated in a resistor is $P = \frac{I}{V}$.
   b. The formula for the power dissipated in a resistor is $P = \frac{V}{I}$.
   c. The formula for the power dissipated in a resistor is $P = IV$.
   d. The formula for the power dissipated in a resistor is $P = I^2V$.

17. What is the formula for power dissipated by a resistor given its resistance and the voltage across it?
   a. The formula for the power dissipated in a resistor is $P = \frac{R}{V^2}$
   b. The formula for the power dissipated in a resistor is $P = V^2R$
   c. The formula for the power dissipated in a resistor is $P = \frac{V^2}{R}$
   d. The formula for the power dissipated in a resistor is $P = I^2R$

Check your Understanding

18. Which circuit elements dissipate power?

a. capacitors
b. inductors
c. ideal switches
d. resistors

19. Explain in words the equation for power dissipated by a given resistance.

a. Electric power is proportional to current through the resistor multiplied by the square of the voltage across the resistor.
b. Electric power is proportional to square of current through the resistor multiplied by the voltage across the resistor.
c. Electric power is proportional to current through the resistor divided by the voltage across the resistor.
d. Electric power is proportional to current through the resistor multiplied by the voltage across the resistor.